1 Derive continuity equation for regolith thickness

$$H(x,y,t) = H_0(x,y) + U(x,y,t) + V(x,y,t,H) \tag{1}$$

Where H_0 is the initial topography,

U is the vertical component of tectonic displacement,

and V is the geomorphic displacement.

Because U is the same for H and B at a given point (x,y), it does not change the regolith thickness:

$$h(x,y) = (H(x,y) - U(x,y)) - (B(x,y) - U(x,y))$$

or

$$h(x,y) = H(x,y) - B(x,y)$$

Therefore, it is convenient to consider soil continuity in terms of soil thickness.

We use the derivation of conservation of transportable (regolith) for our control area (note that is a 2 dimensional problem):

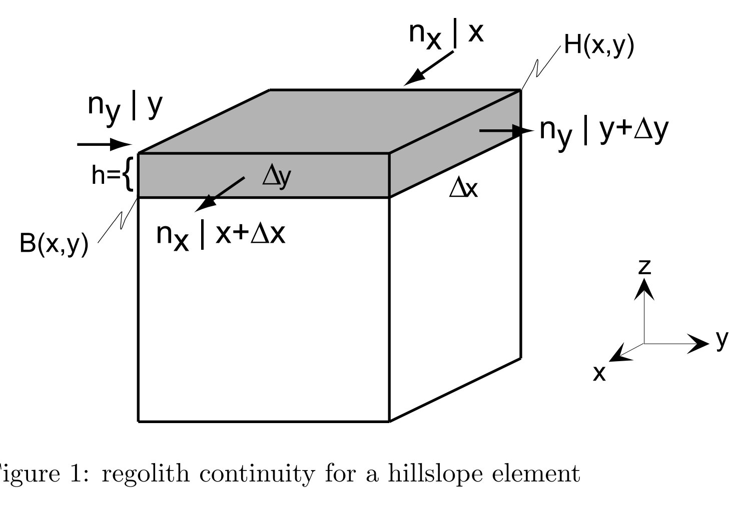

Figure 1: regolith continuity for a hillslope element

$$\Big\{\text{net rate of mass efflux from control area}\Big\} +$$
$$\Big\{\text{net rate of mass accumulation in control area}\Big\} -$$
$$\Big\{\text{rate of production of mass within control area}\Big\}$$
$$= 0$$

So let's take each term and evaluate it:

1. Net rate of mass efflux from control area. Consider the mass flux vector *per unit width*

$$\vec{n} = \rho\vec{v}h = \rho_s\vec{q}_s \qquad [ML^{-3} \cdot L^1T^{-1} \cdot L^1] = [ML^{-3} \cdot L^2T^{-1}] = ML^{-1}T^{-1} \tag{2}$$

where h is the thickness of the regolith or mobile layer (and through which the flux moves) $[L]$

$\vec{v}$ is the sediment velocity $[L]$

ρ_s is sediment density $[ML^{-3}]$

$\vec{q}_s$ is sediment volume flux *per unit width* $[L^2T^{-1}]$

If we resolve the mass flux vector into its cartesian components, we have:

in the x direction: $n_x\Delta y|_{x+\Delta x} - n_x\Delta y|_x$

in the y direction: $n_y\Delta x|_{y+\Delta y} - n_y\Delta x|_y$

in the z direction: $n_z = 0$

2. Net rate of accumulation of mass in the control area:

$$\rho_s\frac{\partial h}{\partial t}\Delta x\Delta y \tag{3}$$

3. Net rate of production of mass in the control area:

$$-\rho_r\frac{\partial B}{\partial t}\Delta x\Delta y \qquad \rho_r \text{ is the rock density} \tag{4}$$

The negative sign ensures that a negative change in B causes positive mass change.

Put it all together:

$$\Big\{(n_x\Delta y|_{x+\Delta x} - n_x\Delta y|_x) + (n_y\Delta x|_{y+\Delta y} - n_y\Delta x|_y)\Big\}+$$
$$\rho_s\frac{\partial h}{\partial t}\Delta x\Delta y-$$
$$-\rho_r\frac{\partial B}{\partial t}\Delta x\Delta y = 0$$

Divide through by $\Delta x \Delta y$ and simplify

$$
\begin{aligned}
(\frac{\Delta n_x}{\Delta x} + \frac{\Delta n_y}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
(\frac{\partial}{\partial x} n_x + \frac{\partial}{\partial y} n_y) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\nabla \cdot \vec{n} + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0 \\
\rho_s \nabla \cdot \vec{q}_s + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} &= 0
\end{aligned}
$$

$$
\rho_s \frac{\partial h}{\partial t} = -\rho_s \nabla \cdot \vec{q}_s - \rho_r \frac{\partial B}{\partial t} \tag{5}
$$

Thus (5) is the continuity equation for regolith thickness.

If we want to go for transport limited conditions, $\rho_r \frac{\partial B}{\partial t} = 0$, and you can divide through by ρ_s:

$$
\frac{\partial h}{\partial t} = \frac{\partial H}{\partial t} = -\nabla \cdot \vec{q}_s
$$

or in 1 dimension:

$$
\frac{\Delta H}{\Delta t} = -\frac{\Delta \vec{q}_s}{\Delta x}
$$

2 Evaluation of dimensions

If we look again at the first portion of (5),

$$(\frac{\Delta n_x}{\Delta x} + \frac{\Delta n_y}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} = 0$$

$$\rho_s(\frac{\Delta q_{sx}}{\Delta x} + \frac{\Delta q_{sy}}{\Delta y}) + \rho_s \frac{\partial h}{\partial t} + \rho_r \frac{\partial B}{\partial t} = 0$$

Focusing on the first term:

$$\rho_s(\frac{\Delta q_{sx}}{\Delta x} + \frac{\Delta q_{sy}}{\Delta y}) = \rho_s \nabla \cdot \vec{q}_s$$

Dimensionally:

$$[ML^{-3}](\frac{[L^2T^{-1}]}{[L]} + \frac{[L^2T^{-1}]}{[L]})$$

$$[ML^{-3}]([L^1T^{-1}] + [L^1T^{-1}])$$

$$[ML^{-3}][L^1T^{-1}] = \rho_s \nabla \cdot \vec{q}_s$$

So, indeed

$$[L^1T^{-1}] = \nabla \cdot \vec{q}_s$$